A10288W1

FIRST PUBLIC EXAMINATION

Trinity Term

Preliminary Examination in Physics

Paper CP2: PHYSICS 2

Tuesday, 7 June 2016, 2.30 pm – 5.00 pm

Time allowed: $2\frac{1}{2}$ hours

Answer **all** *of Section A and* **three** *questions from Section B.*

Start the answer to each question on a new page.

The use of approved calculators is permitted.

A list of physical constants, mathematical formulae and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Moderators expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Section A

1. Three identical charges q each of mass m are fixed in an equilateral triangle of side a. Calculate the work done in assembling this charge distribution by bringing the charges from infinite separation. [2]

If the charges are released simultaneously, describe their subsequent motion and find an expression for their final speeds. [2]

2. Give the expression for the force on a charge q in the presence of both a uniform magnetic flux density $\mathbf{B}$ and a uniform electric field $\mathbf{E}$. If $\mathbf{E} = \mathbf{0}$, describe qualitatively the motion of a particle initially moving at velocity $\mathbf{v}$ (a detailed derivation of the equations of motion is not required, but you should give in your answer expressions for any important speeds, timescales and length scales). [4]

3. Two slabs of charge, extending to large distances in the yz plane, are placed touching each other at $x = 0$ in a vacuum. The first slab has a width of a and a uniform fixed charge density of $\rho = -C/a$ for $-a \leq x < 0$, and the second slab has a width of b and a uniform fixed charge density of $\rho = C/b$ for $0 \leq x < b$, where C is a constant. The $\mathbf{E}$-field outside the slabs is zero. Derive expressions for the electric field, $\mathbf{E}$, within each of the slabs. [5]

What is the electric potential difference, ΔV, between the planes $x = -a$ and $x = b$? [3]

4. A closed circular loop of wire of radius r initially lies in the xy plane. It subsequently rotates about the x-axis through its centre with angular velocity ω. A uniform magnetic flux density, $\mathbf{B}$, is applied along the z-axis. If the wire loop has high resistance R, calculate the *emf* induced in the loop, and the instantaneous current. [3]

Find an expression for the mean time-average torque required to rotate the loop. [3]

5. Write down Ampère's law relating the magnetic flux density, $\mathbf{B}$, to the electric current, giving both the integral and the differential forms of the law. Explain why a *displacement current* must be introduced, and show that this modification leads to one of the Maxwell equations. [5]

6. Explain what the Rayleigh criterion for resolution is in optics. [2]

The lens of the human eye normally has a diameter of about 5 mm. What is the angular diffraction limit of the eye? The image is formed on the retina, which is 17 mm behind the lens and has a surface density of light receptors of 10^5 per square milimetre. Calculate the angular resolution of the eye when viewing objects at large distance, and comment on whether or not the eye is diffraction limited. [5]

7. Draw simple circuit diagrams in which an operational amplifier is used as
(a) an integrator
(b) a differentiator.
In each case find an equation for how the output voltage V_{out} depends on the input voltage V_{in} and any resistors and/or capacitors used in your circuit. [6]

Section B

8. Explain without formal proof how the method of images can be used to solve electrostatic problems involving conducting surfaces. [4]

An infinite conducting sheet exists in the $z = 0$ plane. A point charge of $+Q$ is located above the sheet at co-ordinates $(0, 0, d)$ and a point charge of $-2Q$ is located at $(0, 0, 2d)$. What is the electric field strength, E, at the point just above the conductor and directly under the charges, i.e., at co-ordinate $(0, 0, 0)$? What is the direction of the E-field at this point? [4]

Derive an expression for the electric field strength at a point just above the conducting sheet and a distance x from the origin. What is the surface charge density of the conducting sheet at this point? Use this to calculate the total charge on the plate. [8]

If $Q = 1\,\mathrm{C}$ and $d = 1\,\mathrm{m}$, at what value of x will the E-field drop to zero? Comment on how the E-field strength just above the conducting sheet will vary with distance x in the plane of the sheet when measured at a large distance ($x \gg d$), and justify your answer. [4]

9. Two concentric thin conducting spheres are arranged such that the inner sphere (of radius a) is maintained at potential V_1, and the outer sphere (of radius b) is earthed. Show that the electric field, $\mathbf{E}$, in the air gap between the spheres (i.e., for $a < r < b$) is given by

$$\mathbf{E} = \frac{V_1}{r^2}\frac{ab}{b-a}\hat{\mathbf{r}}.$$

[5]

If $V_1 = 10^3\,\mathrm{V}$, $a = 0.1\,\mathrm{m}$ and $b = 0.4\,\mathrm{m}$, calculate the maximum electric field strength between the two conducting spheres in this two-sphere system. [2]

A third thin concentric conducting sphere with radius c is maintained at potential V_2 and is inserted between the other two spheres (i.e., $a < c < b$). Derive equations for the electric field strength as a function of radius r in the regions $a < r < c$ and $c < r < b$, expressing your answers as a function of V_1, V_2 and the radii of the spheres (a, b and c). [3]

Let $E_{\max}$ be the maximum electric field strength in the region $a < r < b$. Given that the radius of the middle sphere in the three-sphere system is $c = 0.2\,\mathrm{m}$, calculate the value of V_2 which will minimize $E_{\max}$. For this value of V_2, what is the value of $E_{\max}$? Sketch the variation of V with radius in the region $a < r < b$ for both the two-sphere and three-sphere systems. On another plot, sketch the variation of the $\mathbf{E}$-field, and indicate where the $\mathbf{E}$-field is strongest in each case. [10]

10. Use a ray diagram to show that for a real image, the focal length f of a thin convex lens is related to the distance of an object from the lens u and the distance of the image of this object from the lens v by

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$$

What is the magnification of this system? Is the image upright or inverted? [6]

Find an expression for the minimum distance between the object and image in this system. [3]

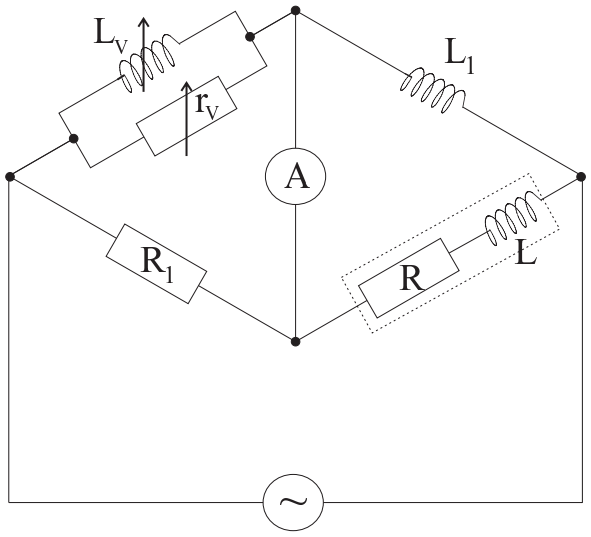
What is meant by *real* and *virtual* images, and what are the conditions for each to occur using a thin convex lens? [4]

An object is placed at a distance D away from a screen, and a thin convex lens is placed between the object and the screen such that a real image is formed. Show that there are two positions of the lens which produce real focussed images on the screen. Find an expression for the distance l between these two positions in terms of D and the focal length of the lens f. [7]

11. (i) Explain the use of complex impedances in circuits driven by a sinusoidal voltage of angular frequency ω. Determine the complex impedances of a resistor, a capacitor and an inductor. [5]

For a circuit, calculate the total complex impedance:
(a) for a resistor R, capacitor C and inductor L connected in series and driven by a sinusoidal voltage of angular frequency ω;
(b) for a resistor R, capacitor C and inductor L connected in parallel and driven by a sinusoidal current of angular frequency ω.
In both cases, give your answer in terms of ω, L, C and R, and give the phase relation between the voltage and the current. At what frequency do the circuits become purely resistive? [5]

(ii) The bridge circuit below is used to determine values of L and R of a non-ideal inductor (which is modelled as an ideal inductor L in series with a resistance R). With the circuit driven by a sinusoidal voltage, the variable inductor L_V and variable resistor r_V are adjusted such that the bridge is balanced, and there is no potential difference across A.



Find expressions for L and R in terms of the other components in the circuit. Give two applications of bridge circuits. [10]