A10287W1

FIRST PUBLIC EXAMINATION

Trinity Term

Preliminary Examination in Physics

Paper CP1: PHYSICS 1

also

Preliminary Examination in Physics and Philosophy

Monday 6th June 2016, 2.30 pm – 5 pm

Time allowed: $2\frac{1}{2}$ hours

Answer **all** *of Section A and* **three** *questions from Section B.*

Start the answer to each question on a new page.

The use of approved calculators is permitted.

A list of physical constants, mathematical formulae and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Moderators expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

In answering any question on this paper you may assume the following formulae for the moments of inertia about the principal axis of a body of mass M, uniform density, and radius a: disk, $\frac{1}{2}Ma^2$; hoop, Ma^2.

Section A

1. A cannon with muzzle velocity v_1 is mounted on top of a tower of height h at a horizontal distance l from a target on the ground. The cannon fires horizontally. A second cannon at the base of the tower fires simultaneously at an angle θ to the horizontal, with muzzle velocity v_2. At what angle should the second cannon fire to allow the two projectiles to collide in mid-air? At what time does this collision occur? [You may ignore air resitance]. [5]

2. Explain what is meant by the *centre of momentum frame*. For the non-relativistic case, show that if kinetic energy is conserved in an elastic collision as viewed from one frame of reference then it is also conserved in all other non-accelerating frames. [6]

3. A train can be considered as a point mass m located at a height h midway between wheels running on two rails separated by a distance d. The train travels at constant speed v around a curve of radius R ($\gg d$). What is the limiting speed v_0 for the train to remain in full contact with the rails? [4]

4. A rocket ejects mass at a constant velocity, relative to the rocket. Obtain an expression for the change in the rocket's velocity that will be obtained as the total mass is reduced from M at launch (from a point in free space) to $M - m$ when the fuel is exhausted. Obtain a new expression for a 2-stage rocket where the initial mass is $M_1 + M_2$, each stage carries a mass m of fuel, and the shell of the first stage is released when the first burn is complete. [8]

5. A ball travels with constant speed u ($\ll c$) into a cylinder of slightly larger diameter which moves with speed v ($\ll u$), along a line at an angle θ to the direction of u. At what angle to the direction of motion must the cylinder be tilted to avoid the ball touching the walls of the cylinder? [4]

6. Now consider the relativistic version of question 5, with light from a distant star reaching the earth from an angle θ to the direction of the Earth's motion. Show that the direction of the star θ' as observed from the earth is given by

$$\cos\theta' = \frac{\cos\theta + \beta}{1 + \beta\cos\theta}.$$

[7]

7. Write down the Lorentz transformations of special relativity. Consider a square laminar object travelling at relativistic speed v in a direction parallel to two of its sides. If the lamina travels past a stationary observer, such that the plane of the lamina lies along the line-of-sight, show that it will appear to the observer to be rotated by an angle $\theta = \sin^{-1}(v/c)$. [*You may assume that the distance to the observer is much greater than the size of the lamina.*] [6]

Section B

8. State the parallel and perpendicular axis theorems. [4]

A uniform disk of radius R, initially of mass M, has a small circular hole of radius a and mass m removed from it at a distance b from the centre. What is the moment of inertia of the resulting object about the centre O of the original disk, and where is the centre of mass located? [6]

The disk is now hung vertically such that it is free to rotate about a horizontal axis through O and perpendicular to the plane of the disk. Derive the Lagrangian for the motion of the centre of mass and find the frequency of small oscillations. [4]

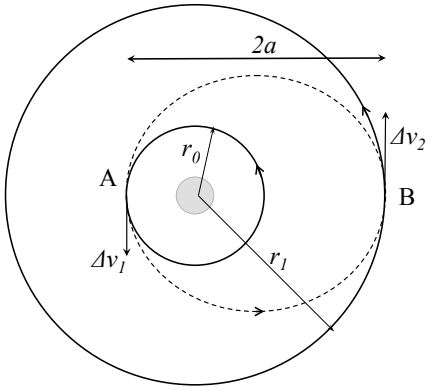
What will be the new frequency of oscillations if the small disk of radius a is added to the opposite side of the disk such that the small disk is within the area of the large disk, touching at a tangent, and the line between the centre of this extra disk and the centre of the hole passes through O? [6]

9. A small mass m is linked to a larger mass M by a chain of length L, mass per unit length ρ, lying across a frictionless table. The system is initially at rest and arranged such that the large mass is suspended just beyond the edge, a vertical distance $y(< L)$ from the floor. The large mass is released and the chain slides off the table, pulling the smaller mass with it.

Determine the Lagrangian of the system, and use this to obtain expressions for the position and velocity of m appropriate to the subsequent times where M is falling and has not yet reached the floor. [10]

The mass m is now attached to a horizontal spring which is fixed to the table, such that M is a distance y_1 from the floor at equilibrium. By considering the Lagrangian, or otherwise, obtain an expression for the frequency of oscillations of the system if M is displaced vertically by a small amount and then released. [10]

10. Consider a satellite of mass m orbiting a planet of mass M. Derive expressions for the orbital velocity v_0 and total energy E_0 for a stable circular orbit of radius r_0. [4]



Assuming that the total energy for an elliptical orbit of semi-major axis a is the same as for a circular orbit of radius a, and the mass of the satellite is constant:

(i) Show that the orbital velocity v_e at a point on such an elliptical orbit when the distance from the planet is r is given by

$$v_e^2 = GM(\frac{2}{r} - \frac{1}{a}).$$ [2]

(ii) At the point A as shown above, determine the instantaneous tangential change in velocity Δv_1 required to change a circular orbit of radius r_0 into an elliptical orbit that just touches another circular orbit of radius r_1 ($r_1 > r_0$). [4]

(iii) At the point B, determine a second instantaneous change in velocity Δv_2 required to transfer the satellite into a new circular orbit of radius r_1. [2]

(iv) Compare the total Δv $(= |\Delta v_1| + |\Delta v_2|)$ required for this transfer to the Δv required for a slow, continuous change of orbital velocity to move from r_0 to r_1. [3]

(v) Now consider the goal of transferring a satellite from a launch vehicle in a circular low earth orbit (altitude 300 km) to geostationary orbit. Compare the total Δv required for this change to that required to extend the elliptical orbit to $r_1 = \infty$ and escape from the Earth. [Take the mass of the Earth to be 5.9×10^{24} kg.] [5]

11. A positron of total energy E collides with a stationary electron to produce three identical uncharged particles each of rest mass M_0. Obtain an expression for the minimum energy E required for this process to occur. [6]

For $M_0 = 135$ MeV/c^2, calculate the speed of the resulting particles in the laboratory frame when the collision occurs at this minimum energy. [8]

One of the particles now decays into a pair of photons. Determine the range of photon energies that could be observed from this process in the laboratory frame. [6]