A10427W1

SECOND PUBLIC EXAMINATION

Honour School of Physics and Philosophy Part A

A2P: ELECTROMAGNETISM

TRINITY TERM 2015

Tuesday, 9 June, 2.30 pm – 4.10 pm

Time allowed: 1 hour 40 minutes

Answer **all** *of Section A and* **two** *questions from Section B.*

For Section A start the answer to each question on a fresh page.
For Section B start the answer to each question in a fresh book.

A list of physical constants and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Examiners expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

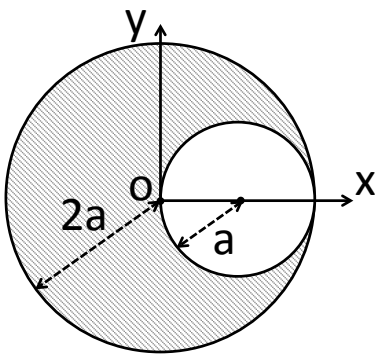
Section A

1. A charge of magnitude q is placed in each of the four corners of a square with side length a. Calculate the electrostatic potential in the centre of the square. [3]

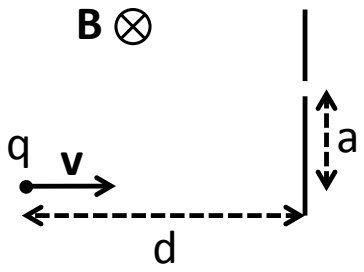
2. State Gauss's law and explain how it may lead to Laplace's equation for the electrostatic potential. [2]

Use Laplace's equation to determine the electrostatic potential inside a parallel-plate capacitor formed of two large, plane metal sheets separated by a distance d in vacuum. [3]

3. A long conducting cylindrical wire has a cylindrical cavity along its length, resulting in the cross-section indicated by the shaded area in the diagram below. The wire carries a current I uniformly distributed over its cross-section. Derive an expression for the resulting magnetic flux density outside the wire. [8]



4. A particle of mass m and charge q enters a region of homogeneous magnetic flux density $\mathbf{B}$ with initial velocity $\mathbf{v}$, as shown in the diagram below.



Determine the magnetic flux density required for the particle to pass through a slit placed a horizontal distance d and a vertical distance a away from its entry point. [6]

5. Show that the relation $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ does not necessarily satisfy conservation of charge. Determine the displacement current to be included to ensure charge conservation. [5]

Section B

6. A charge $+q$ is placed at position $\mathbf{r}_+ = (0, 0, +a)$, and a charge $-q$ is placed at $\mathbf{r}_- = (0, 0, -a)$, where a is a constant. Together, these charges define an electric dipole of moment $\mathbf{p} = q\,(\mathbf{r}_+ - \mathbf{r}_-)$.

Show that the electric potential created by the dipole at a distant point $\mathbf{r} = (r, \theta, \phi)$ in the far-field ($r \gg a$) can be written as

$$V(\mathbf{r}) = \alpha \frac{\cos\theta}{r^2} \ ,$$

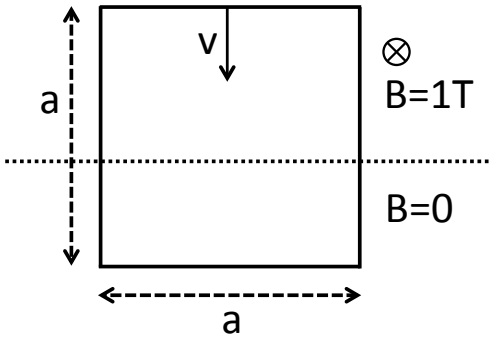
and give an expression for the prefactor α. [6]

Derive the electric field generated by the dipole from its potential in the far-field. Express your result in spherical polar co-ordinates and in cartesian coordinates. [6]

A charge $+q$ is placed a distance d away from a very large, flat metal slab. Derive an expression for the electric field at the surface of the slab that faces the charge, and calculate the total surface charge induced in the metal slab. [8]

7. State the laws of electromagnetic induction. [2]

A vertical square loop of wire with sides of length $a = 8$ cm is falling with velocity v from a region of horizontal magnetic flux density $\mathbf{B}$ of magnitude 1 T into a region where $\mathbf{B} = 0$, as shown in the diagram below. The wire has a diameter d of 1 mm and is made of copper with resistivity $\rho_c = 1.7 \times 10^{-8}\ \Omega$ m and density $\rho_m = 8960$ kg m^{-3}. The resistivity ρ_c of a wire is given by its resistance times the wire cross section divided by its length.



Derive an expression for the current I in the loop as a function of B and v, the resistivity ρ_c, and the loop's dimensions. [5]

Describe and determine the forces acting on each side of the loop, and indicate their directions. [5]

Calculate the magnitude of the steady state velocity v_s, assuming this is reached while the upper arm is still in the magnetic flux density. [8]

8. State Gauss' law in electrostatics in differential form, and show how it can be converted into its integral form. [4]

An infinite, non-conducting sheet of negligible thickness carries a negative uniform surface charge density $-\sigma$. This sheet lies on top of an infinite parallel, non-conducting slab of thickness d carrying a positive uniform volume charge density $+\rho$. All charges are fixed. Calculate the direction and magnitude of the electric field in the following regions:

(a) above the negatively charged sheet;

(b) below the slab;

(c) in the slab. [9]

Sketch the form of the electric field as a function of position along the normal to the slab and sheet, clearly labelling your axes. Assume $\rho d > \sigma$. [7]