A10288W1

FIRST PUBLIC EXAMINATION

Long Vacation

Preliminary Examination in Physics

Paper CP2: PHYSICS 2

Tuesday 8 September 2015, 9.30 pm – 12.00 noon

Time allowed: $2\frac{1}{2}$ hours

Answer **all** *of Section A and* **three** *questions from Section B.*

Start the answer to each question on a fresh page.

The use of calculators is permitted.

A list of physical constants, mathematical formulae and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Moderators expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Section A

1. Write down expressions for the electric field $\mathbf{E}$ and the electrostatic potential V at a distance $\mathbf{r}$ from a point charge of magnitude $+q$. Show that your expressions are consistent with the vector relation $\mathbf{E} = -\nabla V$. [5]

2. A plane electromagentic wave is represented by the following electric field equations:

$$E_x = 0, E_y = A\sin(kx - \omega t), E_z = B\cos(kx - \omega t)$$

Determine the corresponding magnetic flux density vector. [6]

3. A collection of identical filament light-bulbs is to be connected to a battery of zero internal resistance, with the goal of dissipating maximum power. Deduce whether it is preferable to connect the light-bulbs in series or in parallel, explaining your reasoning. [5]

4. State Ampere's law. Derive expressions for the magnetic field around and within a long cylindrical wire carrying current I, assuming that the current is distributed uniformly over the cross-section of the wire. [5]

5. Derive an expression for the capacitance C of two very long, coaxial cylindrical conducting shells with radii R_1 and R_2 and axial length L. [5]

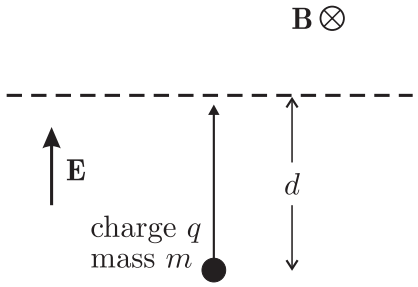
6. Show that the energy stored in an inductor of self-inductance L carrying a current I can be written as $\frac{1}{2}LI^2$. Using the example of a long solenoid, show that the magnetic energy per unit volume associated with a magnetic flux density of magnitude B can be expressed as $\frac{1}{2}B^2/\mu_0$. [7]

7. State Fermat's Principle and use it to derive Snell's law of refraction. [7]

Section B

8. In a particular experiment, a particle of mass m and charge $+q$ moves with speed v along the x-axis towards increasing x. Between $x = 0$ and $x = b$, there is a region of uniform magnetic flux density $\mathbf{B}$ in the y-direction. Deduce the conditions under which the particle will reach the region $x > b$. In the event that it does reach this region, find an expression for the angle to the x-axis at which it will enter it. [10]

In a second experiment, the same particle is accelerated from rest by a constant electric field $\mathbf{E}$ acting over a length d. The particle then encounters a region of constant magnetic flux density $\mathbf{B}$ perpendicular to its velocity, as shown in the figure. Deduce the magnitude $|\mathbf{B}|$ such that the particle will re-enter the region of constant electric field at a distance d from the point at which it left. Assuming this value of $|\mathbf{B}|$, sketch the particle's trajectory in the region of constant magnetic flux density and derive an expression for the time spent there. [10]



9. The circuit shown below contains three resistors, a capacitor, a switch and a battery of negligible internal resistance. Initially the switch is open and the capacitor is uncharged; at time $t = 0$ the switch is closed.

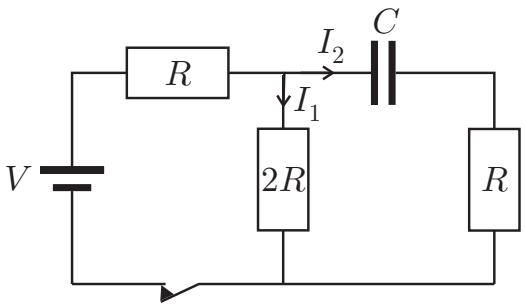
Show that the current I_2 through the capacitor at time t is given by

$$I_2 = A\exp(-t/\tau)$$

and find expressions for A and τ in terms of R, C and V. [11]

Sketch on the same graph I_1 and I_2 as a function of t. [4]

Determine the time at which the currents I_1 and I_2 will be equal, if R=50 kΩ and C=0.2 μF. [5]



10. State Gauss' law in electrostatics in differential form, and show how it can be converted into its integral form. [4]

An infinite, non-conducting sheet of negligible thickness carries a negative uniform surface charge density $-\sigma$. This sheet lies on top of an infinite parallel, non-conducting slab of thickness d carrying a positive uniform volume charge density $+\rho$. All charges are fixed. Calculate the direction and magnitude of the electric field in the following regions:

(a) above the negatively charged sheet;

(b) below the slab;

(c) in the slab. [9]

Sketch the form of the electric field as a function of position along the normal to the slab and sheet, clearly labelling your axes. Assume $\rho d > \sigma$. [7]

11. In a Young's slit experiment, a parallel beam of monochromatic light of wavelength λ=460 nm illuminates two narrow slits, where the slit separation is s, as shown in figure (a). The resulting interference fringes are observed on a screen at a distance d from the slits, where $d \gg s^2/\lambda$. Find an expression for the intensity on the screen as a function of z, the distance along the screen from its centre. [6]

How would this expression change if a focussing lens of focal length f were placed behind the slits, with the screen placed a distance f behind the lens? [2]

Identical transparent cubical boxes of side l=30 mm are placed in front of each of the slits, as shown in figure (b) but with $l \ll d$. The air is gradually removed from one box. The intensity at $z = 0$ is measured as the pressure is reduced to zero. If the number of maxima observed is equal to 20, determine the refractive index of air at atmospheric pressure. [12]

