A10288W1

FIRST PUBLIC EXAMINATION

Trinity Term

Preliminary Examination in Physics

Paper CP2: PHYSICS 2

Tuesday 9 June 2015, 2.30 pm – 5.00 pm

Time allowed: $2\frac{1}{2}$ hours

Answer **all** *of Section A and* **three** *questions from Section B.*

Start the answer to each question on a fresh page.

The use of calculators is permitted.

A list of physical constants, mathematical formulae and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Moderators expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

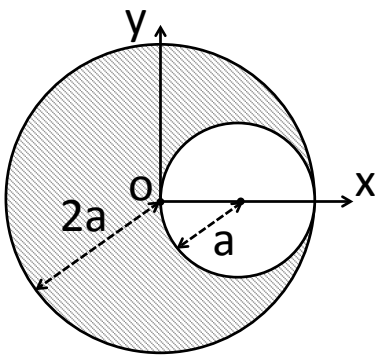
Section A

1. A charge of magnitude q is placed in each of the four corners of a square with side length a. Calculate the electrostatic potential in the centre of the square. [3]

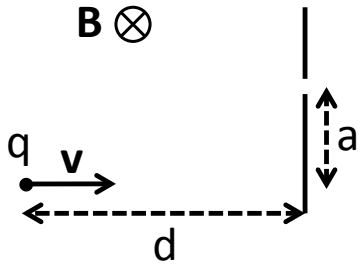
2. State Gauss's law and explain how it may lead to Laplace's equation for the electrostatic potential. [2]

Use Laplace's equation to determine the electrostatic potential inside a parallel-plate capacitor formed of two large, plane metal sheets separated by a distance d in vacuum. [3]

3. A long conducting cylindrical wire has a cylindrical cavity along its length, resulting in the cross-section indicated by the shaded area in the diagram below. The wire carries a current I uniformly distributed over its cross-section. Derive an expression for the resulting magnetic flux density outside the wire. [8]



4. A particle of mass m and charge q enters a region of homogeneous magnetic flux density **B** with initial velocity **v**, as shown in the diagram below.



Determine the magnetic flux density required for the particle to pass through a slit placed a horizontal distance d and a vertical distance a away from its entry point. [6]

5. Show that the relation $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ does not necessarily satisfy conservation of charge. Determine the displacement current to be included to ensure charge conservation. [5]

6. Consider a circuit comprising, in series, a resistor of resistance R, a capacitor of capacitance C, an inductor of inductance L and a switch. At time $t < 0$ the switch is open and the capacitor carries charge Q_0. At time $t = 0$ the switch is closed. Determine the condition for which the subsequent current in the circuit will be oscillatory. [6]

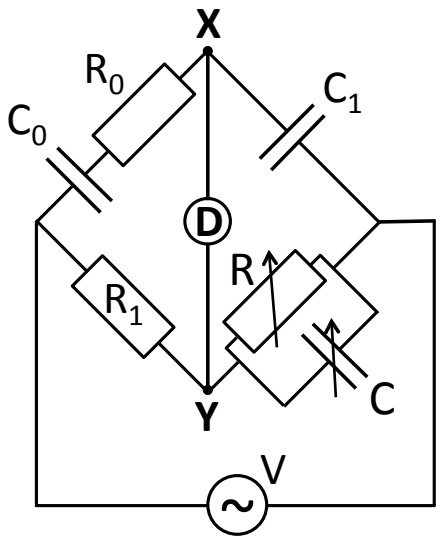
7. A slit of width w is illuminated at normal angle by a collimated beam of coherent light of wavelength λ. Derive the angular intensity distribution in the diffraction pattern and hence determine the Rayleigh (diffraction) limit for a single slit. [7]

Section B

8. Explain what is meant by "complex impedance". Derive expressions for the complex impedances of a resistor, a capacitor and an inductor. [5]

A source of alternating voltage $V_0 \cos(\omega t)$ is connected to a complex impedance $Z = R + iX$. Calculate the time-averaged power dissipated in the circuit, clearly outlining your assumptions and arguments. [5]

A Schering bridge, as shown in the diagram below, is used to determine the capacitance C_0 and resistance R_0 of a lossy capacitor in an AC circuit. For this purpose, the known resistance R and capacitance C are varied until the instrument **D** detects when **X** and **Y** are at the same potential at all times, i.e. the bridge is balanced.



For the case of the balanced bridge, determine expressions for C_0 and R_0 as a function of the other capacitances and resistances in the circuit. [10]

9. A charge $+q$ is placed at position $\mathbf{r}_+ = (0, 0, +a)$, and a charge $-q$ is placed at $\mathbf{r}_- = (0, 0, -a)$. Together, these charges define an electric dipole of moment $\mathbf{p} = q\,(\mathbf{r}_+ - \mathbf{r}_-)$.

Show that the electric potential created by the dipole at a distant point $\mathbf{r} = (r, \theta, \phi)$ in the far-field ($r \gg a$) can be written as

$$V(\mathbf{r}) = \alpha \frac{\cos\theta}{r^2}$$

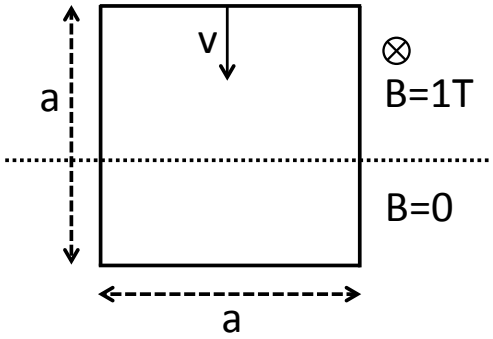
and give an expression for the prefactor α. [6]

Derive the electric field generated by the dipole from its potential in the far-field. Express your result in spherical polar co-ordinates and in cartesian coordinates. [6]

A charge $+q$ is placed a distance d away from a very large, flat metal slab. Derive an expression for the electric field at the surface of the slab that faces the charge, and calculate the total surface charge induced in the metal slab. [8]

10. State the laws of electromagnetic induction. [2]

A vertical square loop of wire with sides a=8 cm is falling with velocity v from a region of horizontal magnetic flux density $\mathbf{B}$ of magnitude 1 T into a region where $\mathbf{B} = 0$, as shown in the diagram below. The wire has a diameter d of 1 mm and is made of copper with resistivity ρ_c=1.7$\times$ $10^{-8}\,\Omega$m and density ρ_m=8960 kg m^{-3}. The resistivity ρ_c of a wire is given by its resistance times the wire cross section divided by its length.

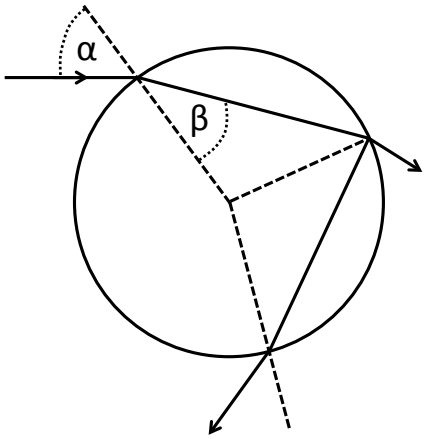


Derive an expression for the current I in the loop as a function of B and v, the resistivity ρ_c, and the loop's dimensions. [5]

Describe and determine the forces acting on each side of the loop, and indicate their directions. [5]

Calculate the magnitude of the steady state velocity v_s, assuming this is reached while the upper arm is still in the magnetic flux density. [8]

11. Sunlight is incident at angle α to the normal to the surface of a spherical raindrop and is refracted at angle β, as shown in the diagram below.



Calculate the total angle δ through which light is deviated, assuming it has experienced a single internal reflection. [6]

Show that the stationary value of δ as a function of α occurs when

$$\cos\alpha = n/A \times \cos\beta,$$

where n is the refractive index of water and A is an integer. State the value of A. [7]

Hence show that when δ is stationary,

$$\sin\beta = \left(\frac{4-n^2}{3n^2}\right)^{\frac{1}{2}}.$$

[5]

Describe how refraction by raindrops can lead to the formation of a rainbow. [2]