A10287W1

**FIRST PUBLIC EXAMINATION**

**Long Vacation**

**Preliminary Examination in Physics**

**Paper CP1: PHYSICS 1**

**also**

**Preliminary Examination in Physics and Philosophy**

**Monday 7th September 2015, 9.30 am – 12.00 pm**

**Time allowed: $2\frac{1}{2}$ hours**

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*Answer* **all** *of Section A and* **three** *questions from Section B.*

*The use of calculators is permitted.*

*A list of physical constants, mathematical formulae and conversion factors accompanies this paper.*

*The numbers in the margin indicate the weight that the Moderators expect to assign to each part of the question.*

**Do NOT turn over until told that you may do so.**

In answering any question on this paper you may assume the following formulae for the moments of inertia about the principal axis of a body of mass $M$, uniform density, and radius $a$:

solid sphere, $\frac{2}{5}Ma^2$; spherical shell, $\frac{2}{3}Ma^2$; disk, $\frac{1}{2}Ma^2$; hoop, $Ma^2$.

## Section A

**1.** Two solid spheres, A and B, each rotate with the same angular frequency about their respective centres. Sphere A has twice the radius and one quarter of the density of sphere B. What is the ratio of their angular momenta? [4]

**2.** A solid wheel of radius $R$ and mass $M$ is composed of a constant density disk of radius $R$ and a thin reinforcing rim which accounts for $\frac{1}{3}$ of the total mass. Show that the moment of inertia about the axis of the wheel is $\frac{2}{3}MR^2$. What is the moment of inertia of the wheel about a perpendicular axis through a tangent point on the rim? [6]

**3.** Starting from conservation of angular momentum in the orbit of a point mass in an elliptical orbit about a massive object, show that the radius vector sweeps out equal areas of the orbit in equal times. [6]

**4.** A ball of mass $M$ moves with relativistic speed $v$ and collides with another ball of mass $3M$ moving in the opposite direction, also with speed $v$. A single object of mass $5M$ is produced in the collision. What is the minimum value of $v$ required for this process to occur? [6]

**5.** A vacuum vessel of volume $1.5\,\mathrm{m}^3$ is fitted with an 800mm diameter glass window which is 80mm thick and has a mass of 110 kg. When the vessel is evacuated, the window has to support standard atmospheric pressure on the outside. If the window shatters, estimate the instantaneous inward acceleration and maximum velocity of the glass fragments, stating your assumptions. [8]

**6.** A student (mass 80 kg) stands at the back end of a punt of mass 150 kg, length 3 m and walks to the other end. Ignoring friction between the punt and the water, how far does the punt move? Discuss whether it would be possible to make use of this motion to move the punt large distances across water. [5]

**7.** Using a space-time diagram, show whether an event $A$ at coordinates $x = 8$ m, $t = 2 \times 10^{-8}$ s could have been caused either by event $B$ ($x = 2$ m, $t = 1 \times 10^{-8}$ s) or by event $C$ ($x = 10$ m, $t = 0.5 \times 10^{-8}$ s). [5]

## Section B

**8.** An object moving at speed $v$ through a medium of density $\rho$ experiences a drag force that can be expressed as $F = C\rho Av^2/2$, where $A$ is the cross sectional area of the object and $C$ is the dimensionless drag coefficient.

A spherical pebble of radius $r$ and density $\rho_p$ falls vertically in still water of density $\rho_w$. Write down the differential equation that describes the descent of the pebble. [4]

Derive an expression for the limiting descent velocity of the pebble in water. Evaluate this limiting velocity for $r = 2\,\text{cm}$, $\rho_p = 2.5\,\text{g}\,\text{cm}^{-3}$ and $C = 0.7$. [6]

The pebble is dropped from a height of 5 m above the surface of a calm lake, which is 5 m deep. Stating clearly any assumptions you make, estimate

(a) How far the pebble travels in the water before reaching its limiting velocity. [5]

(b) How long it takes for the pebble to reach the bottom of the lake from the time it is dropped. [5]

**9.** Derive an expression for the total energy $E$ of a satellite of mass $m$ orbiting a body of mass $M >> m$. Identify and explain each of the terms in your expression. [8]

A satellite is in a circular orbit of radius $R$ about a star of mass $M$. The star ejects 1% of its mass by means of a spherically symmetric wind which removes the mass to a large distance. What are the new nearest and furthest distances of the satellite's orbit around the star? [12]

**10.** A bob of mass $m$ is suspended on a light rod of length $l$ which is fixed at the upper end, but free to pivot in any direction and moves under gravity. The angle between the rod and the downward vertical line is $\theta$, and the azimuthal angle is $\phi$.

(a) Write down the Lagrangian of the system. [4]

(b) Show that the energy and the angular momentum $h$ about the principal axis are constants of the motion. [3]

(c) Derive the equation of motion of the $\theta$ coordinate. [3]

(d) Show how the equation of motion simplifies in the two special cases of constant $\phi$ and constant $\theta$, and derive the angular frequency of the motion in each case. [2]

In the case of approximately constant $\theta \approx \theta_0$, rewrite the equation of motion in terms of the quantity $h$ obtained above, and find an expression for $h$ in terms of $\theta_0$. [3]

Now consider small oscillations in $\theta$, such that $\theta = \theta_0 + \alpha(t)$. Derive the period of small oscillations of $\theta$. [5]

**11.** Write down the Lorentz transformations for energy and momentum. Show that $E^2 - c^2p^2$ is invariant under the Lorentz transformation. [6]

An electron is accelerated through a potential difference until it has a total energy $E$, and collides with a second electron which is initially at rest in the laboratory frame. After the collision, the two particles travel with paths inclined to the original track of the first particle by $\pm\theta$. By considering the motion of the particles in the lab frame, or otherwise, find an expression for $\theta$ in terms of $E$ and $E_0$, the rest energy of the electron. [10]

The angle of deviation of each electron is measured to be 30 degrees. What was the inital accelerating potential? [4]