A10287W1

FIRST PUBLIC EXAMINATION

Trinity Term

Preliminary Examination in Physics

Paper CP1: PHYSICS 1

also

Preliminary Examination in Physics and Philosophy

Monday 8th June 2015, 2.30 pm – 5 pm

Time allowed: $2\frac{1}{2}$ hours

Answer **all** *of Section A and* **three** *questions from Section B.*

The use of calculators is permitted.

A list of physical constants, mathematical formulae and conversion factors accompanies this paper.

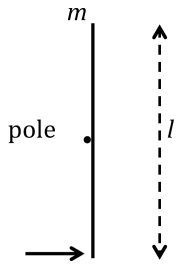
The numbers in the margin indicate the weight that the Moderators expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

Section A

1. You throw a ball with speed v_0 at a vertical wall a distance l away. At what angle to the horizontal must you throw the ball to hit the highest possible point on the wall? What happens when $l > v_0^2/g$? [5]

2. A non-uniform stick of mass m, length l, and moment of inertia $\beta m l^2$ lies on a frictionless horizontal surface. The midpoint of the stick, which is also the centre of mass, touches a thin vertical pole. The stick is struck at one end, as shown below. What is the minimum value of β required for the trailing end of the stick not to hit the pole as it moves away? [5]



3. You are riding on a sled of mass M, moving on flat, frictionless ice, with initial speed v_0. Snow falls onto the sled at a constant mass per unit time, σ. Determine the speed of the sled as a function of time when:

A : You continuously sweep the snow off the sled so that it moves away in a direction perpendicular to the motion of the sled as seen by you.

B : You continuously sweep the snow off the sled so that it moves away in a direction perpendicular to the motion of the sled as seen by an observer stationary on the ice.

C : You do nothing, and let the snow build up on the sled. [7]

4. A projectile of constant mass is launched from the surface of the Earth. Ignoring atmospheric drag, determine the minimum launch speed required for the projectile:

(i) to achieve escape velocity from the Solar System.

(ii) to crash into the Sun. [7]

5. A mass M travelling non-relativistically collides elastically with a stationary mass m and deviates from its original path by an angle θ. Determine an upper limit for θ in the case where $M > m$. [5]

6. An object is moving in frame S' with velocity v_1. S' is moving with speed v_2, in the same direction as v_1, relative to an observer in frame S. State the Lorentz transformations between S' and S and use them to derive an expression for the speed of the object in S. [6]

7. A stationary mass M_A decays into two masses, M_B and M_C. What are the energies and momenta of M_B and M_C? [5]

Section B

8. Consider a point mass moving in one dimension, subject to a smoothly varying potential $V(x)$ and initially at rest in a local minimum of $V(x)$. The mass is given a small impulse such that it moves away from and then back towards the equilibrium. What is the condition on $V(x)$ such that this situation corresponds to simple harmonic motion for suitably small amplitudes? Derive an expression relating the frequency of oscillations about an equilibrium point to a derivative of the potential. [4]

Now consider a point mass m moving with angular momentum L under the action of a radial force such that $V(r) = \beta r^k$.

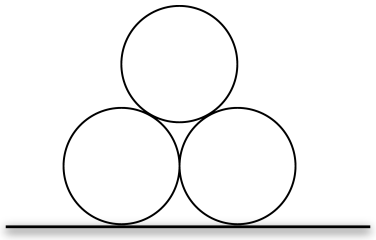
(a) By considering the Lagrangian of this system, find an expression for the energy in terms of r and L, and show that this is a constant of the motion. In this context, explain what is meant by the effective potential. [4]

(b) Derive an expression for the radius r_0 of a circular orbit in terms of β, k, m and L. [4]

(c) The mass is disturbed slightly, such that the radius oscillates about r_0. Find the frequency of these oscillations and the ratio of this frequency to the orbital frequency at r_0. [5]

(d) Use the result from part (c) to describe the shape of the perturbed orbits for $k = 2; k = 7$; and $k = -7/4$. [3]

9. Three identical cylinders, each of mass m, radius R, and moment of inertia $\beta m R^2$, where β is a constant, are arranged in a triangle as shown below.



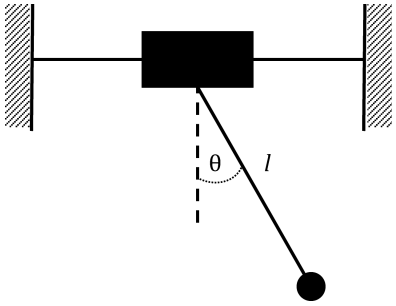
(a) The ground and the cylinders are frictionless. A constant horizontal force is applied to the left cylinder, directed to the right, giving the system an acceleration a. For what range of a will all three cylinders remain in contact with each other? [5]

(b) For the same initial configuration as in (a), but with no external force, determine the initial downward acceleration of the top cylinder when:

(i) There is friction between the ground and the cylinders, but not between the cylinders. [7]

(ii) There is no friction between the ground and the cylinders, but there is friction between the cylinders. [8]

10. A pendulum consists of a mass m and a rigid massless stick of length l. The pendulum is supported from a block mounted on a frictionless horizontal support, as shown below.



(a) Write down the Lagrangian for this system. [2]

The support block is now driven horizontally such that the position of the pivot is given by $x(t) = A\cos\omega t$.

(b) Assuming small angles, derive the solution for the angle of the pendulum as a function of time. [6]

(c) Discuss qualitatively what happens as ω approaches $\sqrt{g/l}$. [2]

The support block, of mass M, is now released and is free to move along the support in either direction. Assuming small angles and velocities:

(d) Show how this situation modifies the Lagrangian from part (a) and use this to find the general solution for the position $x(t)$ of the block and the angle $\theta(t)$ of the pendulum in this case. [8]

(e) Describe the evolution of the system for the case where the block is initially at rest and the mass m has a small initial horizontal displacement. [2]

11. A person runs with speed v into a tunnel of length L. At the instant they enter the tunnel, as measured in the frame of the tunnel, a light source at the far end emits a single photon. Two clocks, one at each end of the tunnel, are synchronised in the frame of the tunnel to read zero when the photon is emitted. The photon is detected by the person when they have travelled a fraction f along the tunnel.

(a) Working in the frame of the tunnel, determine an expression for f. [3]

(b) In the frame of the person:

(i) What is the time on the clock at the tunnel entrance when the photon is emitted?

(ii) What is the distance from the person to the tunnel entrance at this time?

(iii) What is the distance travelled by the photon to reach the person?

(iv) Using your answer for (iii), determine the fraction along the tunnel that the person has travelled in this frame. [12]

(c) The source emits light from a single atomic transition at a rest wavelength of λ_0. Derive an expression for the wavelength observed by the person entering the tunnel. For $\lambda_0 = 589.1$nm find the speed v at which the person entering the tunnel would observe the light at a wavelength of 500.0nm. [5]