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Possible amendments to questions for the open book exam format are indicated in blue.

FIRST PUBLIC EXAMINATION

Trinity Term

Preliminary Examination in Physics

Paper CP1: PHYSICS 1

also

Preliminary Examination in Physics and Philosophy

Monday 3 June 2013, 2.30 pm – 5 pm

Time allowed: $2\frac{1}{2}$ hours

Answer **all** *of Section A and* **three** *questions from Section B.*

Start the answer to each question on a new page.

The use of approved calculators is permitted.

A list of physical constants, mathematical formulae and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Moderators expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

In answering any question on this paper you may assume the following formulae for the moments of inertia about the principal axis of a body of mass M, uniform density, and radius a:

solid sphere, $\frac{2}{5}Ma^2$; spherical shell, $\frac{2}{3}Ma^2$; disk, $\frac{1}{2}Ma^2$; hoop, Ma^2.

Section A

1. ~~State the parallel and perpendicular axis theorems.~~ Describe how the parallel and perpendicular axis theorems can be useful in finding the moments of inertia of solid and planar bodies. Determine an expression for the moment of inertia of a solid cylinder of radius r and length h about an axis through its centre of mass and perpendicular to its principal axis. [5]

2. Explain what is meant by a cyclic co-ordinate and its relationship to a conservation law. ~~Prove~~ Show that energy is conserved for an isolated system with a Lagrangian which has no explicit time dependence, explaining all the steps. [5]

3. A ball of rest mass M and relativistic energy E collides elastically with a stationary ball of rest mass m, such that the direction of motion is along the line joining the centres of the two balls.

(a) Assuming $c = 1$, show that the final energy of M is given by:

$$E' = \frac{2mM^2 + E(m^2 + M^2)}{2Em + m^2 + M^2}$$

(b) Find the limiting value of E' when:

(i) $M >> m$, but $M^2 << Em$

(ii) $m >> M$ [7]

4. Find the maximum distance that can be reached by an arrow fired from a tower of height h with initial speed u, explaining the principles that you use to derive the expression. Ignore air resistance. [5]

5. State the Lorentz transformations for x and t between two reference frames S and S', moving with relative velocity v along the x axis. Use the transformations to show that the contraction of a fixed length (parallel to the x axis) seen in S by an observer moving with S' would be observed as the dilation of the time taken by an object in S' to traverse the same length, as seen an observer moving with S. [6]

6. A high energy particle, travelling towards an observer with velocity v decays to produce a photon with energy E in the rest frame of the particle. ~~Find~~ Explain how you would find the frequency at which this photon would be detected by the observer, and find an expression for it. [6]

7. An alpha particle is accelerated from rest through a potential difference of 20 kV. It travels directly towards a stationary Beryllium nucleus (4 protons, 5 neutrons). Calculate the distance of closest approach. [6]

Section B

8. A satellite orbits at a fixed point above the Earth's equator. Assuming the earth has uniform density, $\rho = 5500$ kg m^{-3}, radius R, and angular frequency of rotation, ω:

(a) Find an expression for η, such that the orbital radius of the satellite is expressed as ηR. [5]

(b) Evaluate the numerical value of η. [1]

The concept of a space elevator relies on a cable rising from a station on the earths surface to geostationary orbit and beyond. Consider a cable, with fixed mass per unit length, which extends radially from just above the surface to end a distance $\eta' R$ from the centre of the Earth (ignore the effects of the atmosphere).

(c) By determining the total force on the cable, or otherwise, Find an expression and numerical value for η'. [10]

(d) At what point in the cable does the tension reach its maximum value? [4]

9. A uniform disk of mass $2M$, radius R, is mounted on a frictionless horizontal pivot through its principal axis. The disk has an additional point-mass, M, fixed to a point on its circumference.

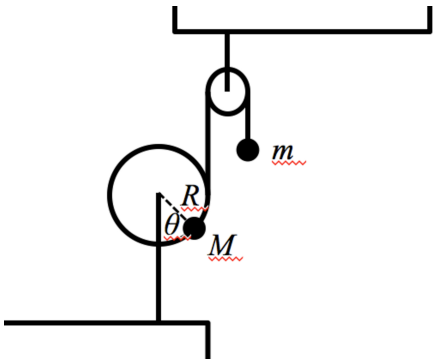
(a) Give the Lagrangian for this system. [2]

(b) Find the frequency of small oscillations of the weight about its lowest point. [3]

The disk is now placed on its rim on a flat surface and set in motion such that it can roll without slipping.

(c) Show that the period of small oscillations has increased by a factor of $\sqrt{3/2}$ relative to the answer obtained in part (b). [8]

A weight, mass M, is attached to the rim of a massless, frictionless pulley, mounted through its axis and linked by a light string to a second weight of mass m ($< M$) suspended from a second massless, frictionless pulley as shown below.



(d) Find the equation of motion of the weight M and the condition for equilibrium of the system in terms of the two masses and the angle θ. [3]

(e) Find the frequency of small oscillations of this system about the point of equilibrium. [4]

10. A spaceship of length 100m in its rest frame is travelling at relativistic speed $0.85c$ towards a space station and sends a signal requesting docking clearance. The station controller, who is unfamiliar with special relativity, identifies the incoming ship and assigns a docking berth appropriate to the length of the ship as measured from the station, with 2m clearance at either end.

(a) What is the length of the docking berth? [3]

(b) What length of the berth as measured from the ship? [3]

When the ship is 1 light day away from dock it launches a message capsule at speed $0.9c$, in the ship's frame, towards the station containing details of its cargo.

(c) How long does it take for the capsule to reach the station:

(i) by the ship's clock,

(ii) by the capsule's clock? [6]

(d) What does the station controller measure as the speed of the capsule? [5]

(e) The ship reduces its speed to $0.7c$ relative to the station. What is the speed of the ship as measured from the capsule? [3]

11. A probe of mass M_p moves in the gravitational potential due to a star of mass M_*.

(a) Write down the Lagrangian for the motion of the probe. [2]

(b) Use this to derive the relationship between the probe's speed and radial position. [4]

The Voyager 1 probe escaped from Saturn (orbital radius 1.5×10^9 km), and moved radially away from the sun with an initial speed of 22 km s^{-1}. Use the relationship obtained above to determine the speed of the probe:

(c) When it reaches a distance of 1.8×10^{10} km from the sun. [2]

(d) In the limit of large distance, where the probe is no longer significantly affected by the gravitational influence of the sun. [1]

Using this limiting speed, consider the probe to be travelling in interstellar space. The probe enters a uniform stationary dust cloud of density ρ:

(e) Assuming that all the dust in the path of the satellite sticks to the forward facing surface, obtain expressions for the speed, mass, and distance of the satellite into the cloud at a later time, t. [8]

(f) The probe takes 30 years to cross this cloud. Determine its final mass and speed, given that $M_p = 722$ kg, $\rho = 10^{-12}$ kg m^{-3}, and the cross-sectional area is 10 m^2. [3]